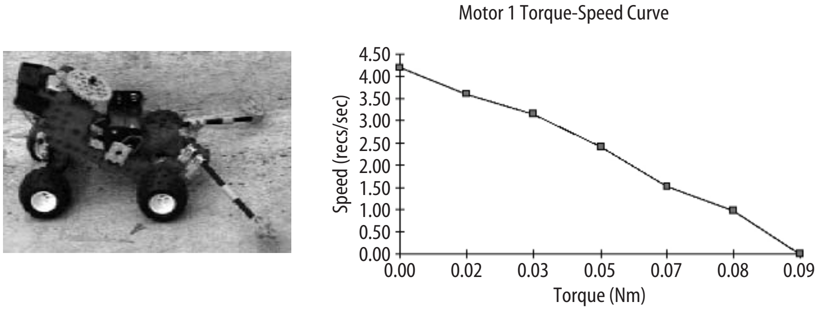
very specific. However, this specific empirical knowledge supports the understanding of concepts. Developing a useful understanding of a formal concept relies on discovering it in several different real contexts. The variety of contexts bounds the concept, establishing its characteristics, limits and conditions of use. The kinds of learning detailed in Table 4.2 – practical limits, methods of connection, causal relations, etc. – fall into a similar set of categories as the errors identified by Miller (1995) in student work.

The conversation presented in Figure 4.8 illustrates the students' level of understanding at the early stages of the project. In the course of the project these students made all the discoveries listed in Table 4.2. In Figure 4.8 the students are discussing how the vehicle could detect when it is on the ramp because they "might need to shift gears to go up the ramp." They have heard of terms such as "stall" and "no-load speed" in their lectures, but these have

Table 4.2. Examples of knowledge acquired in the all-terrain vehicle project

Learning outcome	Examples of specific discoveries made through hardware component integration
Practical limits of use	Motor stops at stall torque limit, current consumption is high, wheels won't turn, motor gets hot. Motor coils physically burn out if run above rated voltage for long.
Characteristics of operation (properties of materials and components)	Motor gets hot at stall. Torque speed curve is linear for permanent DC magnet motor. Voltage supplied by 3V alkaline batteries gradually drops from about 3.2V to about 3V, but batteries lose capacity to supply current.
Methods of connection	Motor must be bolted to chassis to apply reaction torque. Lego spline shaft must be attached to motor shaft concentrically or needs flexible coupling to attach motor shaft to load.
Causal relations	Reversing current reverses permanent DC magnet motor direction. As wheel size increases, motor torque needed increases or gearbox must be adjusted. Lighter vehicle puts lower load on motor, lower forces in gearbox, less friction in gearbox. Lighter vehicle requires less power to move it.
Independent reference variables	Work done moving vehicle load up incline is constant. Load on motor from vehicle (through gearbox) determines motor operating speed and motor torque supplied. Must change vehicle weight or change gearbox to change load at motor. Battery voltage is roughly constant. Current drawn depends on impedance.
Physical quantities	50% losses in LegoTM gearbox due to friction. Particular 3V hobby motor draws about 100mA, supplies torque of 1in oz. Two AA alkaline batteries running 3V motor drawing 100mA last approximately 10 minutes.



1 Carol: We might need to shift gears to go up the ramp. It could need more torque.
Could it sense its wheels don't work? You know when the torque is . . .
2 Alice: You mean at stall. [The team looks impressed.]
3 Sean: You listen in class.
4 Alice: At stall it sucks up a whole lot of current. It could blow a fuse.

Figure 4.8 Building on the hardware repertoire. Conversation of three students designing a model motorized all-terrain vehicle as they work to make sense of abstract representations in the context of prototype hardware.

been used in the context of equations and abstract representations; the lecturer explained the concept of stall using a torque-speed graph and the idea that current consumption increased with torque by using a torque-current graph. However, the students have only just begun to try to apply these concepts in the context of hardware design. If one has never tinkered with or pondered about motors, it takes some thought to grasp the idea of the motor reaching a limit where the motor speed reduces to zero and the motor cannot supply enough torque to drive the load. It takes some effort to link the abstract representations on the board to the performance characteristics of hardware and to the problem at hand, and to internalize the meanings of the abstractions – i.e., to relate them to physical experiences. As the conversation in Figure 4.8 indicates, two of the three students initially had difficulty connecting the concept of stall to the problem at hand.

Notice that in phrase 4 of the conversation in Figure 4.8, Alice uses her knowledge of the characteristics and limits of motors to make a rudimentary design proposal. Knowing that the motor stalls at a certain torque and that the motor draws more current when stalled, she proposes that the motor could blow a fuse in order to activate some means of climbing the ramp. Through the process of identifying the characteristics of components, making design proposals and synthesizing them, students gradually build up their hardware repertoire.

After some conceptual design discussions and consultation with the professor, the group began prototyping. As they developed the prototype they gradually identified the operational characteristics and limits of the motor, using transient representations such as speech and gesture to refer to them and then reconfiguring the hardware to account for what they had learned. They decided to begin with a fixed transmission for simplicity. Working together, Carol and Sean hooked up a motor to their vehicle drive train. Neither had connected up a motor before. They positioned the motor on the chassis, meshing the gear attached to the motor shaft with the larger gear on the vehicle's front axle. As they connected power to the motor, which was